Abacus: Multiplication to Perfection

“Numbers Rule the Universe”

- Pythagoras

Dr. Leanne Grillot

Senior Director of Outreach Services,

American Printing House for the Blind

Learning Objectives

- Explain vocabulary, the rule for positioning subproducts, and the purposes of how a multiplication problem is set on an abacus.
- Evaluate multiplication expressions for their purpose of teaching one digit in the multiplier and any number of digits in the multiplicand.
- Evaluate multiplication expressions to teach multiplication with complementary numbers in subproducts.
- Evaluate multiplication expressions to teach any number of digits in the multiplier and the multiplicand.
- Explain common errors students make when multiplying with an abacus.

Student Prerequisite Skills

- Addition and subtraction with multiple digits using complimentary numbers
- Understanding multiplication and division are inverse operations
- Understanding of multiplication and division within 100

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3rd Grade Math Standards for Multiplication

Critical Area

- Develop understanding of multiplication and division and strategies for multiplication and division **within 100**.
  - Equal-sized groups
  - Arrays
  - Area models

Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations and identify and explain patterns in arithmetic.

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4th Grade Math Standards for Multiplication

Critical Area

- They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers.

Operations and Algebraic Thinking

- Use the four operations with whole numbers to solve problems.
  - Interpret a multiplication equation as a comparison
  - Multiply or divide to solve word problems
  - Solve multistep word problems
  - Multiply 4-digit by 1-digit whole numbers
  - Multiply 2-digit by 2-digit whole numbers

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Multiplication Vocabulary, Placement, and Process

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Multiplication Vocabulary

- Multiplicand: The number being multiplied.
- Multiplier: The number doing the multiplying.
- Product: The answer of the multiplication.

$3 \times 5 = 15$

multiplicand $\times$ multiplier = product

Different Symbols for Multiplication

Multiplication Process

- Multiplication on the abacus is used for multiplicands or multipliers of 2 or more digits.
- Students must know their multiplication times tables (up to 9 × 9) to multiply on the abacus.
- It is a combination of mental math and use of the abacus for computation.

Identify the Parts

$8 \times 7 = 56$

$6 \times 5 = 30$

$3 \times 9 = 27$

$52 \times 20 = 1{,}040$

$83 \times 36 = 2,\ 988$

$1 \times 0 = 0$

$43 \times 6 = 258$

$682 \times 43 = 29{,}326$

$51 \times 27 = 1{,}377$

$1 \times 12 = 12$

Placement

- Multiplication on the abacus is very different from addition and subtraction on the abacus because the whole problem is set up on the abacus.

Setting the Abacus

- The multiplicand is set on the extreme left.
- The multiplier is set towards the right side.
- Placement of the multiplier is the most important step in setting up the problem.
- The product is set in the proper place value in the ones, tens, hundreds, and thousands.

Why That Place?

- Think of your largest 2-digit times 1 digit number.
- 99 × 9
- The largest answer is 891 … a three-digit number.
- We place the multiplier where it will not interfere with the product.

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Partial Products

- All partial products have two digits.
- When it is a ‘one-digit’ product we place a zero in front.
- 2 × 8 is 16 but think one-six as you set the one and then the six.
- 2 × 3 is 6 but think zero-six.

Let’s Set 4 × 2 = 8

- Set the multiplicand, 4, in the extreme left column.
- To determine the position of the multiplier, 2, with the right hand moving from right to left, repeat the problem while the right index finger touches a rod for each digit and the word times.
- When the finger touches the last number of the multiplier, that is where one sets the multiplier or begins to set the multiplier if there is more than one digit (i.e., 26 or 231).

Let’s Set 2 × 46 = 92

- Set the multiplicand, 2, in the extreme left column.
- To determine the position of the multiplier, 46, with the right hand moving from right to left, repeat the problem while the right index finger touches a rod for each digit and the word times.
- When the finger touches the last number of the multiplier, that is where one sets the multiplier or begins to set the multiplier if there is more than one digit (i.e., 26 or 231).

Practice Setting Multipliers

243 × 6

12 × 7

150 × 2

32 × 2

1820 × 5

8 × 7

8 × 63

30 × 68

103 × 66

6 × 892

110 × 602

200 × 206

8 × 35

5 × 231

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Check Your Knowledge

1. True or false: When multiplying on an abacus, all partial products have two digits.

   A. True

   B. False

2. Multiply 5 times 37. The multiplicand 37 is set on columns ____.

   A. 3 and 2
   B. 4 and 3
   C. 5 and 4

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Starting to Multiply

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5 × 3 = 15

- Set the multiplier and multiplicand.
- Right forefinger on the 3 and then moved to the right to wait.
- The left finger checks the 5 of the multiplicand. You think 3 X 5 is one-five.
- Set the 1 of 15 in the next column to the right (tens column) where the right forefinger is waiting.
- Set the 5 of 15 in the next column to the right (units column).
- Clear the 3 of the multiplier.
- Clear the 5 (multiplicand).
- Answer is 15.

2 × 3 = 6

- Set the multiplier and multiplicand.
- Right forefinger on the 3 (then move to the right), left finger checks the 2 of the multiplicand. You think 3 X 2 is zero-six.
- Set the 0 of 06 in the column where the right forefinger is waiting (tens column).
- Set the 6 of 06 in the next column to the right (units column).
- Clear the 3 of the multiplier.
- Clear the 2 (multiplicand).
- Answer is 6.

Start Simply

$4 \times 7 = 28$

$2 \times 3 = 6$

$3 \times 1 = 3$

$3 \times 6 = 19$

$4 \times 9 = 36$

$9 \times 9 = 81$

$3 \times 3 = 9$

$8 \times 4 = 32$

$5 \times 2 = 10$

$3 \times 4 = 12$

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One-Digit Multiplicand and Two or More Digit Multiplier

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8 × 37 = 296

- Set the multiplier and multiplicand.
- Right forefinger on the 7 then slide right; left finger checks the 8 of the multiplicand.
- Set the 5 of 56 where the right forefinger is waiting (tens column).
- Set the 6 of 56 in the next column to the right (units column).
- Clear the 7 of the multiplier.
- Right forefinger on the 3 then slide right; left finger checks the 8.
- Set the 2 of 24 where the right forefinger is waiting (hundreds column).
- Set the 4 of 24 in the next column to the right (tens column).
- Clear the 3 (multiplier) and the 2 (multiplicand).
- Answer is 296.

2 × 35 = 70

- Set the multiplier and multiplicand.
- Right forefinger on the 5 then slide right; left finger checks the 2 of the multiplicand.
- Set the 1 of 10 where the right forefinger is waiting (tens column).
- Set the 0 of 10 in the next column to the right (units column).
- Clear the 5 of the multiplier.
- Right forefinger on the 3 then slide right; left finger checks the 2.
- Set the 0 of 06 where the right forefinger is waiting (hundreds column).
- Set the 6 of 06 in the next column to the right (tens column).
- Clear the 3 (multiplier) and the 2 (multiplicand).
- Answer is 70.

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9 × 54 = 486

- Set the multiplier and multiplicand.
- Right forefinger on the 4 then slide right; left finger checks the 9 of the multiplicand.
- Set the 3 of 36 where the right forefinger is waiting (tens column).
- Set the 6 of 36 in the next column to the right (units column).
- Clear the 4 of the multiplier.
- Right forefinger on the 5 then slide right; left finger checks the 9.
- Set the 4 of 45 where the right forefinger is waiting (hundreds column).
- Set the 5 of 45 in the next column to the right (tens column).
- Clear the 5 (multiplier) and the 9 (multiplicand).
- Answer is 486.

Time for Practice 1-digit by 2-digit

$2 \times 10 = 20$

$3 \times 56 = 168$

$6 \times 66 = 396$

$3 \times 63 = 189$

$4 \times 73 = 292$

$5 \times 24 = 120$

$4 \times 47 = 188$

$7 \times 50 = 350$

$2 \times 87 = 174$

$9 \times 21 = 189$

$7 \times 11 = 77$

$9 \times 99 = 891$

$7 \times 69 = 483$

$3 \times 51 = 153$

$8 \times 49 = 392$

$5 \times 26 = 130$

$2 \times 60 = 120$

$4 \times 44 = 176$

More 1-digit by 2-digit Practice

$2 \times 10 = 20$

$3 \times 56 = 168$

$6 \times 66 = 396$

$3 \times 63 = 189$

$2 \times 72 = 144$

$4 \times 47 = 188$

$7 \times 50 = 350$

$2 \times 87 = 174$

$9 \times 21 = 189$

$8 \times 31 = 248$

$5 \times 35 = 105$

$4 \times 52 = 208$

$5 \times 83 = 415$

$3 \times 71 = 213$

$8 \times 31 = 248$

Check Your Knowledge

1. True or false: In the problem 7 × 52, the first multiplication is 7 times 2.

    A. True

    B. False

2. In the problem 5 times 16: When you multiply 5 times the 1 of 16, the resulting partial product is 05. The 5 of 05 is set on the column ___.

    A. Immediately to the left of the 3 of 30.
    B. Containing the 3 of 30.
    C. Containing the 0 of the 30.

Multiplication with Complementary Numbers

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8 × 34 = 272

- Set 8 and 34 in their proper places.
- Right index finger on the 4 and then slide right. Left finger checks the 8.
- You think 32 as three-two. On the 3rd rod set 3 and move to the next rod to the right and set 2.
- **Clear the 4 of the multiplier.**
- Move your right forefinger to read the next number in the multiplier (3) and then place it to the right.

Finishing 8 × 34 = 272

- With the left forefinger think 3 times 8 (the multiplicand) is 24 or two-four, and on the 3rd rod set a 2.
- Move to the next rod to the right to set the four; 4’s complement of 5 is needed. Set the 5 and clear 1 bead on the same rod.
- Clear the multiplier (3, on the 4th rod) and multiplicand (8) and read your product: 272.

Simple Complementary Numbers

$2 \times 27 = 54$

$4 \times 63 = 252$

$2 \times 75 = 150$

$8 \times 83 = 664$

$8 \times 96 = 768$

$7 \times 67 = 469$

$9 \times 76 = 684$

$8 \times 34 = 272$

$6 \times 27 = 162$

$8 \times 39 = 312$

$5 \times 48 = 240$

$5 \times 59 = 295$

$6 \times 96 = 576$

$5 \times 95 = 475$

$9 \times 65 = 585$

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More Simple Complementary Problems

$6 \times 27 = 162$	$8 \times 95 = 760$	$6 \times 98 = 588$
$8 \times 34 = 272$	$7 \times 36 = 252$	$3 \times 94 = 282$
$7 \times 25 = 175$	$2 \times 51 = 102$	$6 \times 31 = 186$
$7 \times 44 = 308$	$3 \times 94 = 282$	$8 \times 95 = 760$
$3 \times 89 = 267$	$6 \times 32 = 192$	$8 \times 73 = 584$

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Complex Complementary Numbers

$9 \times 58 = 522$

$3 \times 36 = 108$

$9 \times 46 = 414$

$4 \times 26 = 104$

$9 \times 27 = 243$

$8 \times 88 = 704$

$8 \times 28 = 224$

$4 \times 79 = 316$

$6 \times 84 = 504$

$7 \times 59 = 413$

More Complex Complementary Problems

$7 \times 72 = 504$

$8 \times 75 = 600$

$7 \times 45 = 315$

$6 \times 68 = 408$

$8 \times 64 = 512$

$7 \times 49 = 343$

$9 \times 68 = 612$

$9 \times 56 = 504$

$9 \times 23 = 207$

$8 \times 63 = 504$

Check Your Knowledge

1. True or false: The problem 9 times 99 is an example of multiplication with complementary numbers.

   A. True

   B. False

2. Which of the following problems has multiplication with complementary numbers?

   A. 8 times 49
   B. 8 times 73
   C. 8 times 98

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Two-Digit Multiplicand and One-Digit Multiplier

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Multiplicands with Two, Three, or More Digits

- Always multiply each digit of the multiplier (**starting with multiplier's first digit on the right**) by each digit in the multiplicand in the order of the occurrence from **left to right**.
- After each digit in the multiplier is multiplied by **all** digits in the multiplicand, that digit in the multiplier is cleared with the right hand.

60 × 7 = 420

- Set 60 on the extreme left of the abacus.
- Set the multiplier of 7 on the 4th rod. Do not forget to include the 0 in 60 when figuring out where to place the multiplier.
- Move your right index finger to the 3rd rod to get ready.
- The left index finger reads 7 × 6 and you think four-two. On the 3rd rod, set 4 and move to the next rod to the right and set 2.
- Clear the multiplier (7, on the 4th rod) and read your product: 420.

First Part of 21 × 8 = 168

- Set the multiplicand 21 on the extreme left of the abacus.
- Set the multiplier, 8, on the 4th rod.
- The right index finger reads the 8 and then moves to the rod to the multiplier's immediate right, the 3rd rod.
- The left index finger looks at multiplicand 2.
- Since 8 × 2 is 16, one-six, push up one bead on the 3rd rod with the right index finger, then move to the next rod on the right, the tens rod, and set 6.
- **Hold your finger on that rod.**

Second Part of 21 × 8 = 168

- The right index finger holds the same rod for you to begin writing on the same rod the next subproduct.
- We should have the 8 (multiplier) in our working memory.
- Since the left index finger reads the 1 in the multiplicand you think zero-eight, with the right index finger on the 2nd rod, set (add) 0 and move to the rod on the right and set 8.
- You are now finished with the multiplier, so clear the 8 (on the 4th rod) and the multiplicand, 21, before you read your answer. .
- The product is 168.

Simple Multiplication

25 × 3 = 75

34 × 8 = 272

77 × 7 = 539

85 × 2 = 170

37 × 4 = 148

45 × 4 = 180

27 × 5 = 135

49 × 5 = 245

More Simple Multiplication

$42 \times 2 = 84$

$57 \times 4 = 228$

$58 \times 5 = 290$

$35 \times 4 = 140$

$63 \times 6 = 378$

$56 \times 5 = 280$

$32 \times 3 = 96$

$81 \times 8 = 648$

Check Your Knowledge

1. True or false: When setting the problem 78 times 4, there will be four unused columns to the right of the 4.

    A. True

    B. False

2. If you multiply 42 times 7___.

    A. The last digit of the product will be found in the unit's column.
    B. There will be four digits in the answer.
    C. The first time you multiply 7, you multiply it by the 2 of 42.

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Two-Digit Multiplicand and Two-Digit Multiplier

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12 × 83 = 996 (Simple Example)

- Set 12 on the extreme left.
- Set multiplier, 83, on fifth and fourth rods
- Right hand on the 3 of 83, then move to rest it on the column to the right.
- Left hand on the 1 of the 12; we say, the 3 of 83 times the 1 of 12 is 03
- Set the 0 of 03 where the right forefinger is resting.
- Move right and set the 3 of the 03 .
- We hold the right forefinger on the 3 while the left hand moves onto the 2 of 12.
- Ready to multiply the 3 (which should have been stored in our working memory) of the 83 by the 2 of 12, which results in 06.

Next Set of Directions for 12 × 83 = 996

- The 0 of the 06 is set on the same rod as the 3 of the 03.
- Move to the right and the right forefinger sets the 6 of 06.
- Since the 3 of the 83 in the multiplier has been multiplied by all the digits in the multiplicand, the right hand clears the 3 of 83.
- Next digit in the multiplier is the 8 of 83. The right forefinger reads the 8 and then moves to the right. Left forefinger moves back to the 1 of 12. We say 8 times 1 is 08.
- The right forefinger sets the 0 of 08 by tapping the rod. Then move right and set the 8 of 08.
- **Hold your finger on that rod!**

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Finishing 12 × 83 = 996

- The left forefinger moves from the 1 of the 12 to the 2; we say 8 times 2 is 16 (zero-eight).
- The 1 of 16 is to be set where the right forefinger is waiting (on the 8).
- The right hand move the right again to set the 6 of 16 (no the 3).
- Since the 8 of 83 has been multiplied by all the digits in the multiplicand, the right hand clears the 8. The left hand clears the 12.
- The product is 996.

45 × 67 = 3015 (Complex Example)

- Set 45 on the extreme left.
- Set multiplier, 67, on fifth and fourth rods
- Right hand on the 7 of 67, then move to rest it on the column to the right.
- Left hand on the 4 of the 45; we say, the 4 of 45 times the 7 of 67 is 28
- Set the 2 of 28 where the right forefinger is resting.
- Move right and set the 8 of the 28.
- We **hold the right forefinger on the eight** while the left hand moves onto the 5 of 45.
- Ready to multiply the 7 (which should have been stored in our working memory) of the 67 by the 5 of 45, which results in 35.

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Next Set of Directions for 45 × 67 = 3015

- The 3 of the 35 is set on the same rod as the 8 of the 28. Since we cannot set the 3 on the same rod as the 8, we set a ten on the column to the left (where the 2 is) and clear 7 (the complementary number of 3).
- Move to the right and the right forefinger sets the 5 of 35.
- Since the 7 of the 67 in the multiplier has been multiplied by all the digits in the multiplicand, the right hand clears the 7 of 67.
- Next digit in the multiplier is the 6 of 67. The right forefinger reads the 6 and then moves to the right. Left forefinger moves back to the 4 of 45. We say 6 times 4 is 24.
- The right forefinger sets the 2 of 24. Then move right and set the 4 of 24 on the same rod as the 3.
- Since the 4 cannot be added to the 3 directly, the right hands sets 5 and clears 1 (complementary number of 4). **Hold your finger on that rod!**

Finishing 45 × 67 = 3015

- To designate the placement of the first digit in the next step of the multiplication, the right hand is held on the rod where the 7 is located.
- The left forefinger moves from the 4 of the 45 to the 5; we say 6 times 5 is 30 (three-zero).
- The 3 of 30 is to be set on the same rod as the 7 where are right forefinger is waiting.
- Since the 3 cannot be set, or added, directly on the same rod, set a ten, and then clear the 7 (complement of 3).
- Both hands move the right again to set the 0 of 30.
- To set the 0 on the same rod as the 1, the right hand touches the rod gently.
- Since the 6 of 67 has been multiplied by all the digits in the multiplicand, the right hand clears the 6. The left hand clears the 45.
- The product is 3105.

Simple 2-Digit by 2-Digit Multiplication

17 × 34 = 578

56 × 20 = 1,120

11 × 82 = 902

53 × 40 = 2,210

75 × 90 = 6,750

90 × 75 = 6,750

64 × 17 = 1,088

89 × 13 = 1,157

74 × 21 = 1,554

35 × 18 = 630

Complex 2-Digit by 2-Digit Multiplication

98 × 98 = 9,604

88 × 59 = 5,192

67 × 81 = 5,427

38 × 65 = 2,470

18 × 85 = 1,530

82 × 74 = 6,068

91 × 22 = 2,002

89 × 35 = 3,115

36 × 45 = 1,620

77 × 59 = 4,543

Check Your Knowledge

1. True or false: The problem 57 × 25 is an example of multiplication with complementary numbers.

   A. True

   B. False

2. Which of the following problems has multiplication without complementary numbers?

   A. 18 times 71
   B. 18 times 21
   C. 31 times 18

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Common Mistakes in Multiplication on the Abacus

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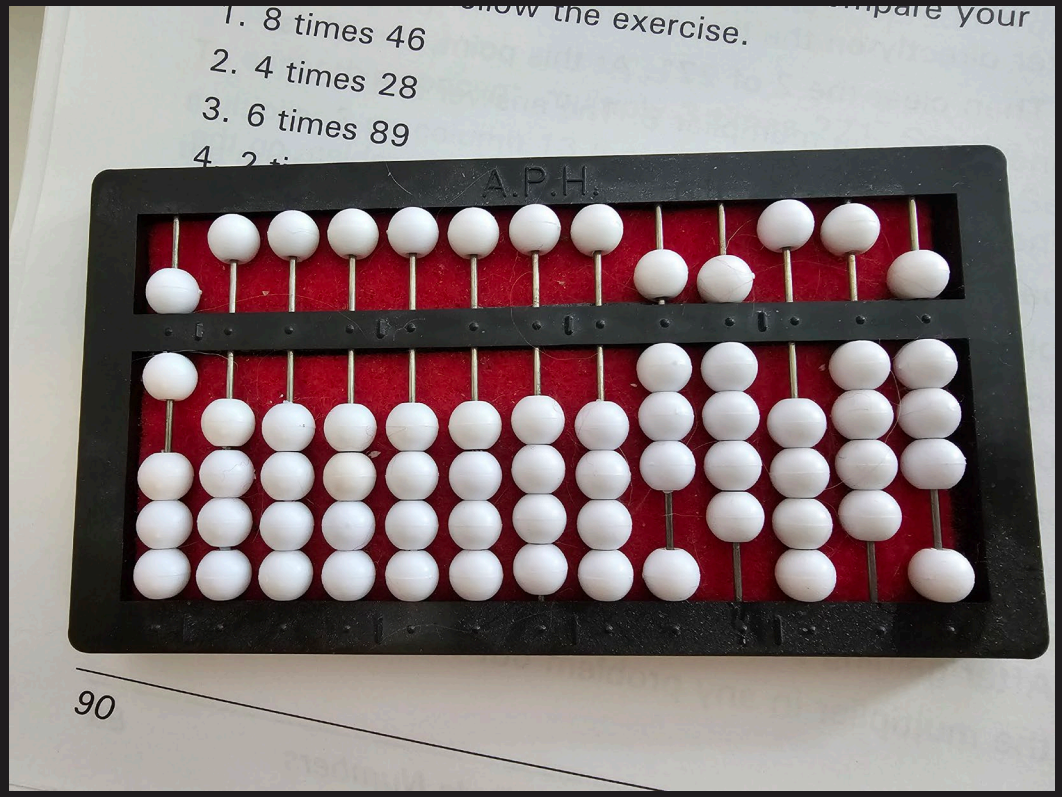
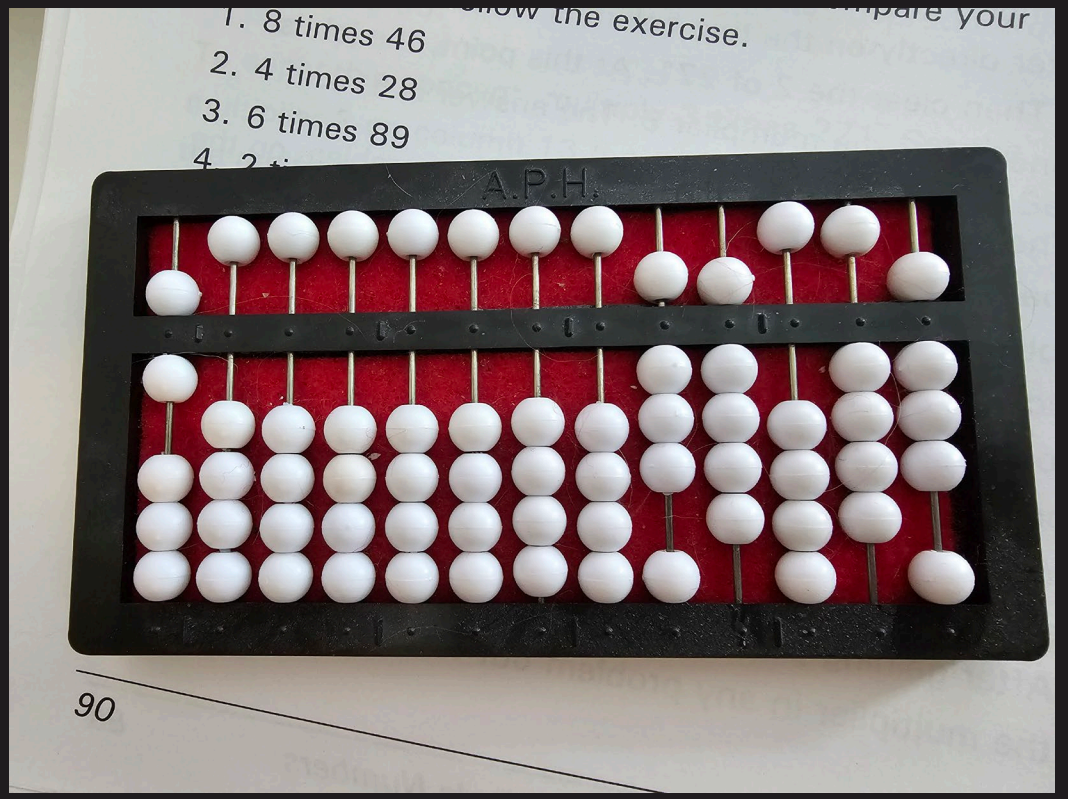
Abacus Errors

- Placement of the multiplicand
- Multiplying the wrong digits together
- Forgetting to give zero a space
- Having subproducts only one digit long
- Not clearing the multiplier or multiplicand



Math Errors

- Wrong numbers
- Multiplication facts
- Missing an entire multiplication step



Games with Numbers

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Quick Pick

- Counting, Addition, subtraction, multiplication, division
- Large print and braille
- Four choices

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Math Flash

- Amazon Alexa, Android
- Self-voicing electronic flash cards
- Addition, subtraction, multiplication, and division

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Math Robot

- iPhone or iPad
- Adults can set the range of numbers, operations, number of tries, etc.
- Low vision mode
- VoiceOver support
- Braille support

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SlapStack Math

- iOS devices
- Addition – displays addition problems with sums from 2-12.
- Subtraction – subtraction problems with differences from 1-11.
- Multiplication – multiplication problems with products from 1-12.
- Division – division problems, using numbers up to 24, with quotients from 1-12.
- Addition and Subtraction – a mixture of problems.
- Multiplication and Division – a mixture of problems.
- All Four Operations – a mixture of problems from all operations.

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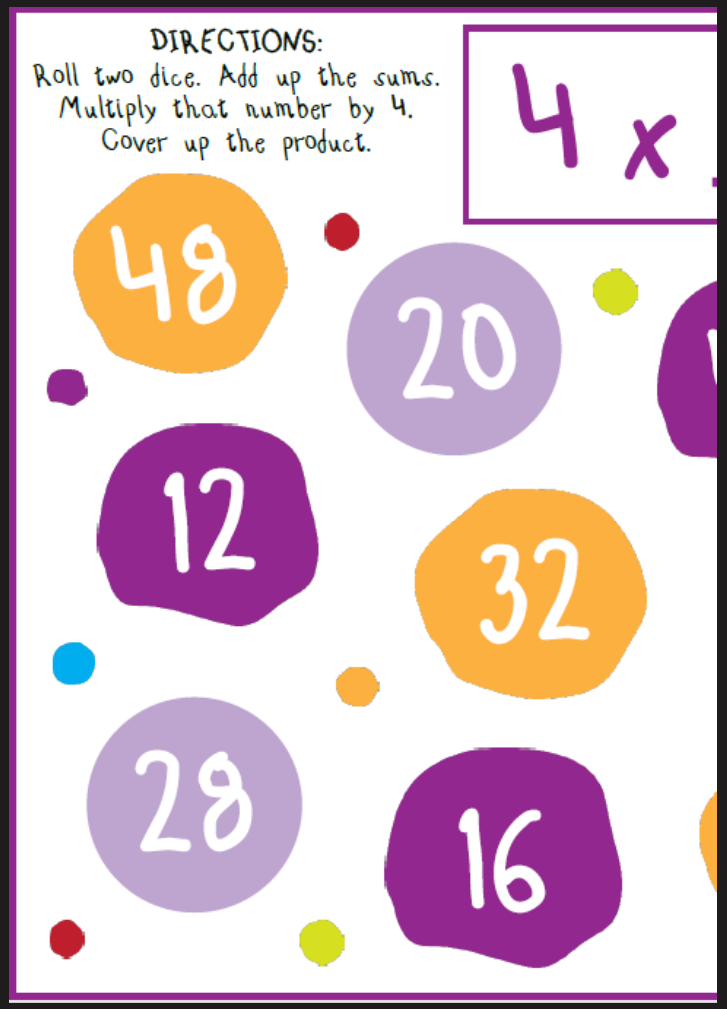
Roll and Bump!

- Roll two dice.
- Add up the sums.
- Multiply that number by 4 (or any other number).
- Cover up the product.
- Opponent's turn. If they get the same product, they can "bump."
- Player with the most covered products wins after ten minutes.



Animal Watch VI Suite

- iPad
- Recommended ages: 5th to 7th grade (math level)
- Accessible via Apple's VoiceOver screen reader
- Features built-in feature that emulates VoiceOver ability to enlarge content using Zoom Built-in scratch pad
- Scaffolded hints to support students in solving word problems
- Engaging information about the animal species

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Sudoku

- Improves logical thinking
- Improves number skills
- Improves decision-making
- Improves memory and recall
- Increases spatial reasoning
- Helps one relax
- Reduces overthinking
- Builds a leisure activity

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Check Your Knowledge

1. True or false: Having partial products with only one digit is a common abacus mistake.

   A. True

   B. False

2. Which of the following games offers practice of addition, subtraction, multiplication, and division?

   A. Sudoku
   B. Roll and Bump
   C. SlapStack Math

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Lesson Plans for Multiplication

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Write A Multiplication Lesson Plan

Here is your scenario:

- You have taught your 4th grade student how to add and subtract using complementary numbers on the abacus over the last two years. He/she has mastered this and is now ready to advance to the next level of instruction.
- Your student already knows their multiplication and division facts through 100.
- Your job is to create a lesson plan that introduces the concept of multiplication to this student.
- Remember, this is an introductory lesson – it should not contain all the information found in this abacus course regarding multiplication; keep in mind the student's age.
- You must decide of how much information to cover.

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ABACUS: MULTIPLICATION TO PERFECTION

Leanne Grillot, lgrillot@aph.org

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