## Abacus: Divide and Conquer

"Numbers Rule the Universe"

- Pythagoras


## Dr. Leanne Grillot

Senior Director of Outreach Services,
American Printing House for the Blind


## Learning Objectives

- Reproduce vocabulary when teaching division using an abacus.
- Explain the rules for quotient figure placement and the purposes of how a division problem is set on an abacus.
- Explain common errors students make when dividing with an abacus.
- Evaluate division expressions to teach single digit divisor division problems without remainders.
- Evaluate division expressions to teach single digit divisor division problems with remainders.
- Explain the process of teaching division with more than one-digit divisors using the partial quotients method on a braillewriter.
- Demonstrate how to use the abacus as a calendar.


## Prerequisite Skills

- Addition and subtraction with multiple digits using complimentary numbers
- Understanding multiplication and division are inverse operations
- Understanding of multiplication and division within 100



## 3rd Grade Math Standards for Division

## Critical Area

- Develop understanding of multiplication and division and strategies for multiplication and division within 100.
- Equal-sized groups
- Arrays
- Area models


## Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations and identify and explain patterns in arithmetic.


## 4th Grade Math Standards for Division

## Critical Area

- Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends.


## Operations and Algebraic Thinking

- Use the four operations with whole numbers to solve problems.
- Interpret a multiplication equation as a comparison
- Multiply or divide to solve word problems
- Solve multistep word problems
- Use place value understanding and properties of operations to perform multi-digit arithmetic.
- 4-digit dividends and 1-digit divisors
- Whole numbers and remainders


## 5th Grade Math Standards for Division

## Critical Area

- Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They develop fluency in these computations and make reasonable estimates of their results.


## Operations and Algebraic Thinking

- Perform operations with multi-digit whole numbers and with decimals to hundredths.
- 4-digit dividends and 2-digit divisors
- Divide decimals to the hundredths


# Division Vocabulary, Placement, and Process 

## Division

- Dividend: The number to be divided.
- Divisor: The number by which the dividend is divided.
- Quotient: The result of division.
- Remainder: The amount left over after the completion of the division because the groups would not be equal.
- Inverse operation: Multiplication reverses division.
$60 \div 20=30 \mathrm{R} 0$
(dividend $\div$ divisor $=$ quotient + remainder)
$60=20 \times 30+0$
(dividend $=$ divisor $\times$ quotient + remainder)

Different Symbols for Division


## 「

## Division Process

- Division on the abacus is used for dividends of 2 or more digits.
- Students must know their division facts (up to $81 \div 9$ ) to divide on the abacus.
- It is a combination of mental math and use of the abacus for computation.


## Identify the Parts

- $3 \div 3=1$
- $11 \div 3=5$ R1
- $63 \div 9=7$
- $22 \div 5=4$ R2
- $848 \div 16=53$
- $111 \div 10=11$ R1


## Practice Setting Division

- $243 \div 6$
- $12 \div 7$
- $153 \div 2$
- $32 \div 2$
- $1820 \div 5$
- $839 \div 7$
- $63 \div 8$
- $37 \div 6$
- $103 \div 4$
- $892 \div 5$
- $110 \div 6$
- $200 \div 2$
- $854 \div 3$
- $1545 \div 9$


## Placement

- Much like multiplication, the entire division problem is set on the abacus.
- Dividend: extreme right of the abacus
- Divisor: extreme left of the abacus
- Quotient: towards the left of the dividend

NOTE: the quotient is not written on rods of equivalent place value. One empty rod is to the right of the quotient. The empty rod stands for the $R$ in remainder. A remainder as large as the devisor will be to the right of the empty rod.

## Check Your Knowledge

1. True or false: The whole division problem is set on the abacus.
A. True
B. False
2. The divisor is placed $\qquad$ .
A. On the far left of the abacus.
B. On the far right of the abacus.
C. Toward the left of the abacus.

$$
\begin{aligned}
& 2 \div 1 \text { Digit Division, } \\
& 3 \div 1 \text { Digit Division, } \\
& \text { and } \\
& 4 \div 1 \text { Digit Division }
\end{aligned}
$$

## Quotient Placement

- Determining where to place the quotient:
- Compare the first number of the divisor if more than one number, with the first number of the dividend.
- Rules
- If the divisor is equal to (SAME) or smaller than the first digit in the dividend, skip one rod to the left, and hold that rod. Think: Same = SKIP
- If the divisor is larger (Not the same) than the first number of the dividend, place your finger to the immediate left of the first number in the dividend. Hold that rod. Think: Larger= LEFT
- You must compare each time before you divide.


## Multiplication in Division $80 \div 8=10$

- Once the partial quotient is placed, we multiply the quotient with the divisor and subtract the result.
- All multiplication answer have two place values.
- When multiplying eight times one, we must subtract 08.
- You subtract a zero in the first column and then the eight from the next column to the right.


## $92 \div 4=23$

- Set dividend 92, extreme right
- Set divisor 4, extreme left
- Compare! Is divisor 4 the same or smaller than the 9 ?
- Answer: Yes; skip a rod and place finger on 4th rod.
- Divide: 4 into 9,2 times, so set 2 where your finger is.
- Multiply: the 2 in the partial quotient times the 4 of the divisor, which is 8 , or 08 .
- Subtract: 08. The zero is subtracted from the 3rd rod, then the 8 is subtracted from the 9 . Now 12 is remaining on the abacus.


## Complete $92 \div 4=23$

- Compare! Is the divisor 4 the same or smaller than 1?
- Answer: No, it is larger, place finger to immediate left (larger = left).
- Divide: 4 into the 12 and set the 3 on the 3 rd rod.
- Multiply: (mentally) 3 times 4 equals 12.
- Subtract: the 12 by subtracting the 1 from the tens rod and the 2 on the units rod.
- Quotient: shows 23 followed by two empty rods. There are no remainders.
- Read: 23 Remainder 0


## 2-Digit Dividends

- $28 \div 7=4$
- $6 \div 3=2$
- $3 \div 1=3$
- $18 \div 6=2$
- $4 \div 2=2$
- $81 \div 9=9$
- $9 \div 3=3$
- $32 \div 4=8$
- $10 \div 5=2$
- $45 \div 9=5$


## $420 \div 7=60$

- Set 7 on the extreme left of the abacus. Set the dividend of 420 .
- Move your right index finger to the 4 of 420 to get ready.
- Compare! The left index finger reads 7. is it the same or smaller than 4.
- Answer: No. Place your finger on the immediate left, the 4th rod, and set 6.
- Multiply: Mentally, 7 times 6 equals 42.
- Subtract: 42 by subtracting the 4 on the hundreds rod and the 2 on the tens rod.
- Clear the multiplier (7) and read your product: 420 followed by two empty rods. There are no remainders.
$910 \div 7=130$
- Set dividend; Set divisor
- Compare! Is divisor 7 the same or smaller than the 9?
- Answer: Yes; $\underline{\text { skip }}$ a rod and place finger on immediate left.
- Divide: 7 into 9 once, so set 1 on the $5^{\text {th }}$ rod.
- Multiply: the 1 in the quotient times the 7 of the divisor, which is 7 .
- Subtract: the 7 from the $3^{\text {rd }}$ rod. Now you have 210.
- Compare! Is divisor 7 the same or smaller than the 2.
- Answer: No, it is larger! Move finger to immediate left.


## Complete $910 \div 7=130$

- Divide: the 7 into 21 and set the 3 on the $4^{\text {th }}$ rod.
- Multiply: Mentally, the 7 times the 3 equals 21 .
- Subtract: 21 by subtracting the 2 on the hundreds rod and the 1 on the tens rod.
- Clear the divisor (7).
- Quotient: 130 followed by two empty rods. There are no remainders.


## 3-Digit Dividends

- $442 \div 2=221$
- $255 \div 5=51$
- $532 \div 7=76$
- $912 \div 6=152$
- $396 \div 2=198$
- $684 \div 9=76$
- $708 \div 3=236$
- $747 \div 9=83$
- $448 \div 3=112$
- $240 \div 5=48$
- $474 \div 3=158$
- $270 \div 9=30$


## 4-Digit Dividends

- $2,438 \div 2=1,219$
- $4,865 \div 5=973$
- $4,836 \div 6=806$
- $8,370 \div 9=930$
- $7,494 \div 6=1,249$
- $3,205 \div 5=641$
- $9,848 \div 8=1,231$
- $5,614 \div 7=802$
$-8,727 \div 3=2,909$
- $6,360 \div 6=1,060$
- $1,840 \div 8=230$
- $270 \div 9=30$

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## Check Your Knowledge

1. True or false: The quotient is not written on rods of equivalent place.
A. True
B. False
2. Divide 515 by 5 . After the first complete round of division, multiplication, and subtraction, the dividend shows $\qquad$ .
A. 35
B. 45
C. 15

## Division with Complementary Numbers

$$
104 \div 7=? ? ?
$$

- Set dividend; Set divisor
- Compare! Divisor 7 with the 1 in the dividend.
- Answer: 7 the larger than the 1 so move to the immediate left.
- Divide: 7 into 10 once, so set 1 on the $4^{\text {th }}$ rod.
- Multiply: the 1 in the quotient times the 7 of the divisor, which is 07 .
- Subtract: The 07 is subtracted from the 10 . You now have 34 remaining in the dividend.
- Compare! Divisor 7 with the 3 of the dividend.
- Answer: 7 is larger than the 3, so move to the immediate left.
- Divide: 7 into 34 and set the 4 on the $3^{\text {rd }}$ rod.
- Multiply: mentally, 7 times 4 equals 28.
- Subtract: 28 by subtracting the 2 on the tens rod and the 8 on the units rod.
- Quotient: 14 separated by one empty rod from the remainder. 14 r6


## 3-Digit Dividends with Complementary Numbers

- $828 \div 6=138$
- $243 \div 9=27$
- $782 \div 2=391$
- $561 \div 3=187$
- $308 \div 2=154$
- $288 \div 8=36$
- $616 \div 7=88$
- $204 \div 6=34$
- $702 \div 3=234$
- $852 \div 6=142$


## 4-Digit Dividends with Complementary Numbers

- $8,120 \div 8=1,015$
- $5,008 \div 8=626$
- $6,980 \div 4=1,745$
- $8,085 \div 7=1,155$
- $4,914 \div 3=1,638$
- $3,040 \div 8=380$
- $1,148 \div 7=164$
- $1,251 \div 3=417$
- $4,524 \div 6=337$
- $2,359 \div 7=337$


## Check Your Knowledge

1. True or false: In the problem $4,644 \div 6$, the remaining dividend after the first complete round of division, multiplication, and subtraction is 344 .
A. True
B. False
2. When solving $962 \div 8$,
A. The first quotient figure is sent on the first column to the left of the dividend.
B. The first quotient figure is set on the second column to the left of the dividend.
C. The second quotient figure is set on the second column to the left of the dividend.

## Division with Remainders

## $93 \div 4=23 R 1$

- Set dividend, extreme right
- Set divisor, extreme left
- Compare! Is divisor 4 smaller than the 9 ?
- Answer: Yes; skip a rod and place finger on 4th rod.
- Divide: 4 into 9,2 times, so set 2 where your finger is.
- Multiply: the 2 in the quotient times the 4 of the divisor, which is 8 , or 08.
- Subtract: 08. The zero is subtracted from the 3rd rod, then the 8 is subtracted from the 9 . Now 13 is remaining on the abacus.


## Complete $93 \div 4=23$ R 1

- Compare! Divisor 4 is larger than 1. Place finger to immediate left.
- Divide: 4 into the 13 and set the 3 on the 3rd rod.
- Multiply: (mentally) 3 times 4 equals 12
- Subtract: the 12 by subtracting the 1 from the tens rod and the 2 on the units rod.
- Quotient: shows 23 followed by one empty rod and then 1. There is a remainder of 1.
- 23 R 1


## $2,349 \div 7=335 R 4$

- Set dividend; Set divisor
- Compare! Divisor 7 with the 2 in the dividend.
- Answer: 7 is larger than the 2 , so start in immediate left.
- Divide: 7 into 23 , so set 3 on the $4^{\text {th }}$ rod.
- Multiply: the 3 in the quotient times the 7 of the divisor, which is 21 .
- Subtract: The 7 is subtracted from the 23 . You now have 244 remaining in the dividend.


## Next Steps 2,349 $\div 7=335$ R 4

- Compare! Divisor 7 with the 2 of the dividend.
- Answer: 7 is larger than the 2, so move to the immediate left.
- Divide: 7 into 24 and set the 3 on the $3{ }^{\text {rd }}$ rod.
- Multiply: mentally, 7 times 3 equals 21.
- Subtract: 21 by subtracting the 2 on the tens rod and the 1 on the units rod. You now have the remaining 39 in the dividend.


## Complete $2,349 \div 7=335$ R4

- Compare! Divisor 7 with the 3 of the dividend.
- Answer: 7 is larger than the 3, so move to the immediate left.
- Divide: 7 into 39 and set the 5 on the $3^{\text {rd }}$ rod.
- Multiply: mentally, 7 times 4 equals 35.
- Subtract: 35 by subtracting the 2 on the tens rod and the 8 on the units rod.
- Quotient: 335 separated by one empty rod from the remainder. 335 R4


## Practice Simple Remainders

- $7 \div 3=2$ R 1
- $3 \div 2=1$ R 1
- $7 \div 2=6$ R 1
- $98 \div 5=19$ R 3
- $58 \div 9=6$ R 6
- $88 \div 9=9$ R 7
- $14 \div 5=2$ R 4
- $93 \div 7=13$ R 2
- $38 \div 4=9$ R 2
- $87 \div 7=12$ R 3


## Division with Remainders and Complementary Numbers

- $30 \div 7=4 R 2$
- $23 \div 6=3 R 5$
- $57 \div 4=14$ R 1
- $14 \div 9=1$ R 5
- $30 \div 4=7$ R 2
- $11 \div 3=3$ R 2
- $35 \div 4=8$ R 3
- $51 \div 9=5$ R 6
- $25 \div 3=8$ R 1
- $68 \div 8=8$ R 4


## 3-Digit Dividends with Remainders

- $575 \div 8=71$ R 7
- $313 \div 4=78$ R 1
- $647 \div 4=161$ R 3
- $791 \div 6=131$ R 5
- $237 \div 6=39$ R 3
- $780 \div 7=111$ R 3
- $588 \div 5=117$ R 3
- $388 \div 5=77$ R 3
- $436 \div 4=145$ R 1
- $407 \div 6=67$ R 5


## Check Your Knowledge

1. True or false: In the problem $1,586 \div 4$, the remainder is 3 .
A. True
B. False
2. Solve $3,851 \div 7$ on the abacus. The answer is
A. 550 R 1
B. 550 R 2
C. 551 R 1

## Division by a 2-Digit Divisor

## $48,286 \div 25=1931$ R 11

- Set dividend and divisor
- Compare the first two digits of each! Is divisor 25 smaller than the 48 ?
- Answer: Yes; skip a rod and place finger on millions column.
- Divide: 25 into 48, 1 time, so set 1 where your finger is.
- Multiply: the 1 in the quotient times the 2 of the divisor, which is 2 , or 02 .
- Subtract: 02 . The zero is subtracted from the 6th rod, then the 2 is subtracted from the 4 . Leave your finger there!
- Multiply: the 1 in the quotient times the 5 of the divisor, which is 5 , or 05.
- Subtract: 05 . The 0 is subtracted from where your finger is resting (the 2 ), then the 5 is subtracted from the 8 . Now we have 23,286 remaining.


## Next Step: $48,286 \div 25=1931$ R 11

- Compare the first two digits of each! Is divisor 25 smaller than the 23?
- Answer:25 is larger than 23. Move left.
- Divide: 25 into 232, 9 times, so set 9 where your finger is.
- Multiply: the 9 in the quotient times the 2 of the divisor, which is 18.
- Subtract: 18 . The 1 is subtracted from the 5 th rod, then the 8 is subtracted from the 4 th rod. Leave your finger there!
- Multiply: the 9 in the quotient times the 5 of the divisor, which is 45 .
- Subtract: 45. The 4 is subtracted from where your finger is resting (the 5), then the 5 is subtracted from the 3 rd rod (2). Now we have 786 remaining.


## Continue: $48,286 \div 25=1931$ R 11

- Compare the first two digits of each! Is divisor 25 smaller than the 78?
- Answer: Yes. 25 is smaller than 78. skip a rod.
- Divide: 25 into 786, 3 times, so set 3 where your finger is.
- Multiply: the 3 in the quotient times the 2 of the divisor, which is 06 .
- Subtract: 06 . The 0 is subtracted from the 4 th rod, then the 6 is subtracted from the 3rd rod. Leave your finger there!
- Multiply: the 3 in the quotient times the 5 of the divisor, which is 15.
- Subtract: 15 . The 1 is subtracted from where your finger is resting (the 1), then the 5 is subtracted from the 2nd rod (8). Now we have 36 remaining.


## Complete: $48,286 \div 25=1931$ R 11

- Compare the first two digits of each! Is divisor 25 smaller than the 36 ?
- Answer: Yes. 25 is smaller than 36. skip a rod.
- Divide: 25 into 36, 1 time, so set 1 where your finger is.
- Multiply: the 1 in the quotient times the 2 of the divisor, which is 02.
- Subtract: 02. The 0 is subtracted from the 3 rd rod, then the 2 is subtracted from the 2 nd rod (3). Leave your finger there!
- Multiply: the 1 in the quotient times the 5 of the divisor, which is 05.
- Subtract: 05 . The 0 is subtracted from where your finger is resting (the 1 ), then the 5 is subtracted from the 1st rod (6).
- The quotient reads 1,931 R 11


## Partial Quotient Method with a Braillewriter

## Partial Quotients Long Division

- A partial products long division problem has a quotient located at the bottom.
- The vertical line is brailled
- Note spaces on either side of the vertical line.
- Numbers are aligned

| 6962 <br> $\frac{600}{362}$ <br> $\frac{180}{182}$ <br> 180 <br> 2 | 100 |
| ---: | :---: |
| 160 R 2 |  |

 by place value.

## Partial Quotients: UEB, Print, or Nemeth



| $3 5 \longdiv { 4 7 1 }$ |  |
| ---: | ---: |
| $\frac{350}{121}$ | 10 |
| $\frac{70}{51}$ | 2 |
| $\frac{35}{16}$ | $\frac{1}{13}$ R 16 |


| : $:$ |  |
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## Check Your Knowledge

1. True or false: Division with two-digit divisors cannot be completed on an abacus.
A. True
B. False
2. When solving $367 \div 14$ using the partial quotient method:
A. You multiply the divisor with a number.
B. You compare the first digit of the divisor with the first digit in the dividend.
C. You never have remainders.

## Calendar Mathematics

## Code Assigned to Days of the Week

| Number | Day |
| :--- | :--- |
| 1 | Monday |
| 2 | Tuesday |
| 3 | Wednesday |
| 4 | Thursday |
| 5 | Friday |
| 6 | Saturday |
| 7 or 0 | Sunday |

## Basic Formula

- Take the last two digits of the year.
- Divide by four (discard remainders).
- If that year is a leap year, subtract 1 at this time.
- Divide by seven.
- Remainder is the day of the week by Week Codes.


## What Day is January 1, 2030

- Want to know the day of the week January 1, 2030, will fall.
- 30 divided by four. (keep the 7 and discard the remainder [2])
- Add last two numbers of year to quotient. $30+7=37$
- Divide sum by seven. $37 \div 7=5$ remainder 2
- Compare 2 to the Week Codes = Tuesday


## What Day is December 26, 2020

- Want to know the day of the week December 26, 2020, occurred.
- 20 divided by four. $20 \div 4=5$
- Subtract the leap year from the quotient. 5-1 = 4
- Add last two numbers of year to difference. $20+4=24$
- Divide sum by seven. $24 \div 7=3$ remainder 5
- Compare 6 to the Week Codes = Saturday


## Advanced Formula

Need three codes:

- Day Code
- Month Code
- Year Code



## Code Assigned to Days of the Week

## Number

## Day

1
2
3
4
5
6
7 or 0

Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
Sunday

## Code Assigned to Months of the Year

| Month | Number | Mnemonic |
| :--- | :--- | :--- |
| January | $6^{*}(5)$ | W-I-N-T-E-R has 6 letters. |
| February | $2^{*}(1)$ | February is the $2^{\text {nd }}$ month of the year. |
| March | 2 | March 2 the beat of the drum! |
| April | 5 | A-P-R-I-L and F-O-O-L-S have 5 letters. |
| May | 0 | May I have a sandwich? Hold the May - 0! |
| June | 3 | June B-U-G has 3 letters. |

## End Months of the Year

| Month | Number | Mnemonic |
| :--- | :--- | :--- |
| July | 5 | Watching FIVER-works and FIVER-crackers! |
| August | 1 | August begins with A, the $1^{\text {st }}$ letter. |
| September | 4 | September is the beginning of F-A-L-L. |
| October | 6 | Halloween T-R-I-C-K-S and T-R-E-A-T-S |
| November | 2 | I'll have 2 servings of turkey, please! |
| December | 4 | December is the L-A-S-T month, or X-M-A-S. |

## Year Code Formula

- Take the last two digits of the year.
- Divide by 4.
- Discard remainders.
- Add last two digits of the year.
- Divide by 7.
- Add year from table.
- The remainder is the year code.

| YEAR | ADD |
| :---: | :---: |
| 1800 | 3 |
| 1900 | 1 |
| 2000 | 0 |
| 2021 | 5 |

## Steps for Advanced Formula

- Take the last two digits of the year.
- Divide by 4.
- Discard remainders.
- Add last two digits of the year.
- Divide by 7.
- The remainder is the year code.
- Add year from table (none needed for this century).
- Add the month code.
- Add the date.
- Divide by 7.
- Remainder is the day of the week.


## What day of the week was

## May 2, 2007

- Determine Year code: $07 \div 4=1+7=8 \div 7=1$ R. 1
- Month code + Date + Year code $=0+2+1=3$
- 3 = Wednesday


## September 9, 2008 (leap year)

- Determine Year code: $08 \div 4=2+8=10 \div 7=1$ R. 3
- Month code + Date + Year code = 4 + 9 + $3=16$
- Divide by 7 , we have $16 \div 7=2$
- 2 = Tuesday


## January 16, 2008 (leap year)

- Determine Year code: $08 \div 4=2+8=10 \div 7=1$ R. 3
- Month code + Date + Year code = 5 + 16 + $3=24$
- Divide by 7 , we have $24 \div 7=3$
- 3 = Wednesday


## What day of the week?

## March 19, 2061

- Determine Year code: $61 \div 4=15+61=76 \div 7=10$ R. 6
- Month code + Date + Year code $=2+19+6=27$
- $27 \div 7=6$
- $6=$ Saturday


## December 7, 1998

- Determine Year code: $98 \div 4=24+98+1=123 \div 7=17$ R. 4
- Month code + Date + Year code $=4+7+4=15$
- $15 \div 7=1$
- 1 = Monday


## Check Your Knowledge

1. True or false: Using the advanced method to determine a date in 2035, the year code is 1 .
A. True
B. False
2. What is the Month code + Date + Year code of March 3, 1879, before dividing by seven to determine the day of the week?
A. 3
B. 8
C. 11

## Lesson Plan Division

## Write A Division Lesson Plan

Here is your scenario:

- You have taught your 4rd grade student how to add and subtract using complementary numbers on the abacus two years ago. He/she has worked a few weeks on multiplication and needs to advance to the next level of instruction-beginning division.
- Your job is to create a lesson plan that introduces the concept of division to this student.
- Remember, this is an introductory lesson - it should not contain all the information found in this abacus course regarding division; keep in mind the student's age.
- You decide of how much information to cover.


## ABACUS: DIVIDE AND CONQUER

Leanne Grillot, Igrillot@aph.org

